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Synchronization of Multi-character Nazarimehr System using Active Control

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Abstract— The active control consists of the compensation of non-linearities and the decoupling of the equations that describe the dynamics of a system to achieve its stabilization. In this paper, this strategy is applied to a slave system to compel it to follow the dynamics of an autonomous master system. Both systems are based on a new chaotic system recently reported [1]. The parameters of both master and slave systems are assumed to be a priori known. Once the synchronization error between these two systems is established, the corresponding dynamics of this error is calculated. Next, a control law is designed to guarantee the exponential convergence to zero of such error. The performance of this controller is illustrated by numerical simulation.

Index Terms— active control, chaos synchronization, master – slave configuration, error dynamics, Lyapunov analysis

1. INTRODUCTION

CHAOS theory deals with the study of systems with peculiar characteristics such as very high sensitivity to initial conditions, boundedness of solutions, and a rich dynamic behaviour. The current importance and influence of this theory can be verified by the development of useful applications in disciplines as diverse as biology, engineering [2], [3], politics [4], finances [5],[6], administration [7], medicine [8], psychology [9], geophysics [10] and hydrology [11].

The chaotic systems can be represented by means of nonlinear ordinary differential equations of third order or higher order. Although both the structure and the parameters of these equations can be completely a priori known, due to high sensitivity to the initial conditions, the dynamic behaviour of these kind of systems cannot be predicted [12], [13], [14], [15]. In spite of this fact, it is still possible to control and also synchronize them [16].

Synchronization consists of the modification of the dynamic behaviour of two or more systems in such a way that all of them provide the same response simultaneously. The synchronization between two systems can be unidirectional or bidirectional [17], [18]. Unidirectional synchronization refers to the establishment of a master – slave relationship, where the slave system must follow the response of the master system.

The synchronization of chaotic systems can be realized by means of different control strategies. For example, in [19], nonlinear control is utilized to achieve the synchronization between Genesio and Rössler systems. In other papers, controllers based on stability analysis of Lyapunov have been developed [20], [21]. The synchronization with non-linear feedback has also been studied, where systems whose feedback consists of a linear term and a compensation of non-linear terms of each state [22], [23]. On other hand, when the parameters of the systems are unknown, adaptive control can be applied [24]. Other control techniques have also been developed to achieve the unidirectional synchronization of different systems [25], [26], [27], [28], [29].

The active control is a strategy which permits the synchronization of a master-slave configuration in a systematic way. This strategy consists of the compensation of non-linearities and the decoupling of the equations of the error dynamics in such a way that the exponential convergence to zero of the synchronization error can be guaranteed if the parameters are a priori known [30].

With this method, different chaotic systems have been synchronized [31], [32], [33], [34], [35], [36]. Also, a comparison between this method and backstepping control has even been proposed in [37]. Both present an excellent performance when they are implemented in embedded systems.

In this paper, we propose for the first time the synchronization of a master-slave configuration based on a new chaotic system recently reported in [1] by using active control. The control law here proposed is tested by means of numerical simulation.

2. MULTI – CHARACTER CHAOTIC SYSTEM

The system presented in [1] can be described by the following set of ordinary differential equations:

$$\begin{aligned}\dot{x} &= -ay\\ \dot{y} &= bwz + d\\ \dot{z} &= y^2 - cz^2 + \bar{e}\\ \dot{w} &= x + y - z(y + w)\end{aligned} \tag{1}$$

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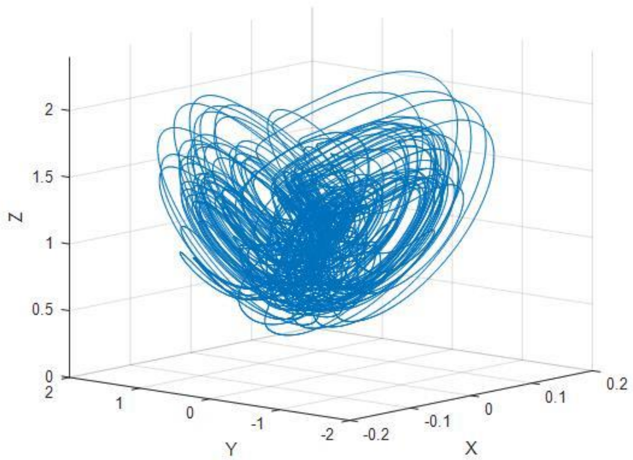

Fig. 1. 3D phase portrait of xyz states

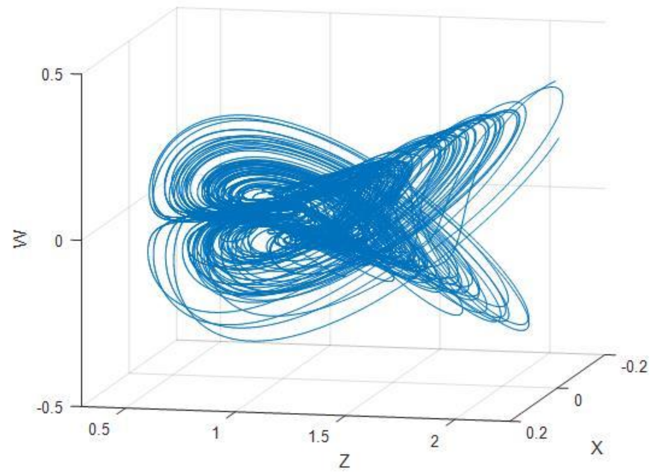

Fig. 3. 3D phase portrait of xzw states

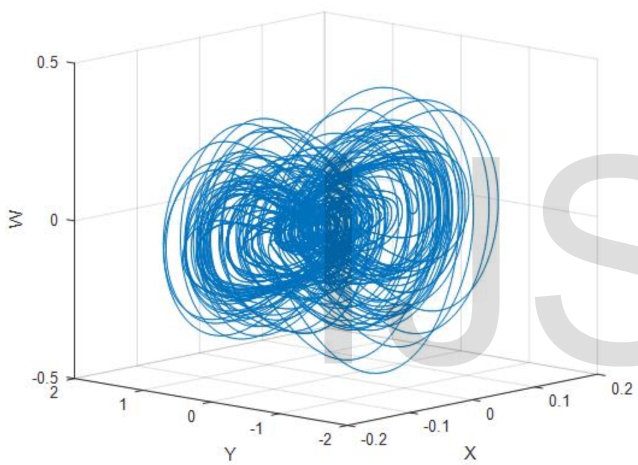

Fig. 2. 3D phase portrait of xyw states

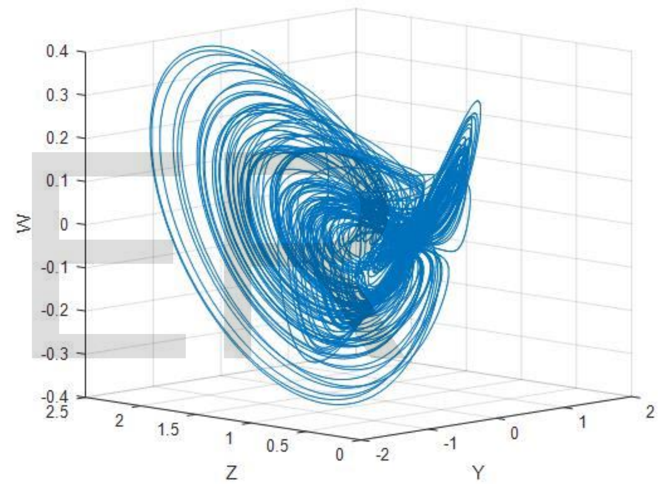

Fig. 4. 3D phase portrait of yzw states

where x, y, z and w are the states and a, b, c, d, y $\bar{e}$ are constant parameters. As the parameters are changed, different dynamic behaviours can be observed. In particular, according to [1], when the parameters are: $a = 0.05$, $b = 5$, $c = 0.28$, $d = 0.1$, and $\bar{e} = 0.01$, with (1,1,1,1) as first initial condition, chaotic regime can be observed. This is illustrated by means of figures 1-4. In these figures, the different 3D phase portraits were obtained by using Simulink® 9.0 with **ode23t solver** and a relative tolerance value of 1e-7.

The high sensitivity of the system (1) to initial conditions is shown in figures 5 - 8. The evolution of each state is presented and compared when the initial condition is changed from (1,1,1,1) to (1.1,1.1,1.1,1.1). In spite of this slight difference, as can be appreciated from these figures the responses diverge.

3. Synchronization via Active control

In this section, by means of active control, a controller is designed to achieve the synchronization between a master system and a slave system both based on Nazarimehr system (1). Taking this fact into account, the master system can be simply described as follows:

$$\begin{aligned}
\dot{x}_m &= -ay_m \\
\dot{y}_m &= bw_m z_m + d \\
\dot{z}_m &= y_m^2 - cz_m^2 + \bar{e} \\
\dot{w}_m &= x_m + y_m - z_m(y_m + w_m)
\end{aligned} \tag{2}$$

And the controlled slave system is given by:

$$\begin{aligned}
\dot{x}_s &= -ay_s + u_1 \\
\dot{y}_s &= bw_s z_s + d + u_2 \\
\dot{z}_s &= y_s^2 - cz_s^2 + \bar{e} + u_3 \\
\dot{w}_s &= x_s + y_s - z_s(y_s + w_s) + u_4
\end{aligned} \tag{3}$$

where u_1, u_2, u_3, and u_4 are the control inputs. It should be noted that the corresponding parameters of the two systems have the same value. The synchronization error between systems (2) and (3) can be defined as:

$$\begin{aligned} e_1 &= x_s - x_m \\ e_2 &= y_s - y_m \\ e_3 &= z_s - z_m \\ e_4 &= w_s - w_m \end{aligned} \tag{4}$$

Thus, the first derivative of (4) is given by:

$$\begin{aligned} \dot{e}_1 &= \dot{x}_s - \dot{x}_m \\ \dot{e}_2 &= \dot{y}_s - \dot{y}_m \\ \dot{e}_3 &= \dot{z}_s - \dot{z}_m \\ \dot{e}_4 &= \dot{w}_s - \dot{w}_m \end{aligned} \tag{5}$$

By substituting (2) and (3) into (5) and after some algebraic operations, the following is obtained:

$$\begin{aligned} \dot{e}_1 &= -ae_2 + u_1 \\ \dot{e}_2 &= b(z_s e_4 + w_m e_3) + u_2 \\ \dot{e}_3 &= e_2(y_s + y_m) - ce_3(z_s + z_m) + u_3 \\ \dot{e}_4 &= e_1 + e_2(1 - z_s) - e_3(y_m + w_m) - e_4 z_s + u_4 \end{aligned} \tag{6}$$

By following the principle of active control, that is, to compensate non-linearities and decouple the equations of error dynamics (6), the following control law can be obtained:

$$\begin{aligned} u_1 &= ae_2 - k_1 e_1 \\ u_2 &= -b(z_s e_4 + w_m e_3) - k_2 e_2 \\ u_3 &= -e_2(y_s + y_m) + ce_3(z_s + z_m) - k_3 e_3 \\ u_4 &= -e_1 - e_2(1 - z_s) + e_3(y_m + w_m) + e_4 z_s - k_4 e_4 \end{aligned} \tag{7}$$

where k_1, k_2, k_3, and k_4 are control gains which permit control the speed of convergence process. If the control law (7) is substituted into (6), then the closed-loop synchronization error dynamics is given by:

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4 \end{aligned} \tag{8}$$

Or well,

$$\dot{e}_i = -k_i e_i, \quad i = 1,2,3,4 \tag{9}$$

The stability of (8) is analysed in the following section.

4. Stability Proof

The stability of system (8) can be determined by means of Lyapunov analysis. Let the Lypunov function candidate be:

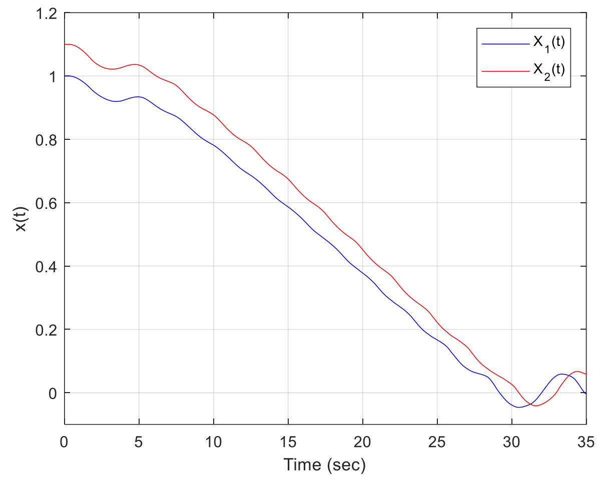

Fig. 5. Time series for $x(t)$ with different initial conditions

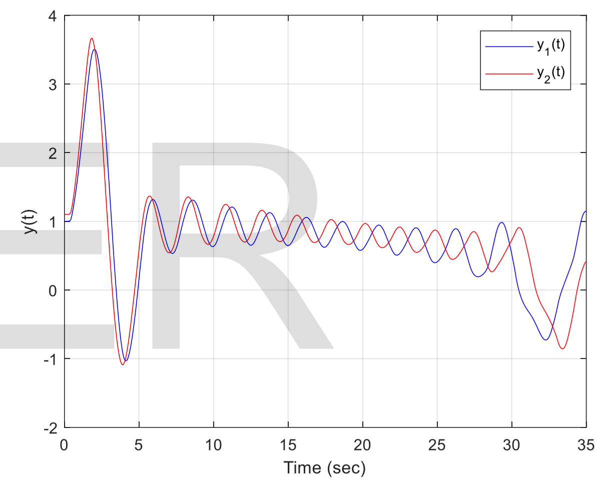

Fig. 6. Time series for $y(t)$ with different initial conditions

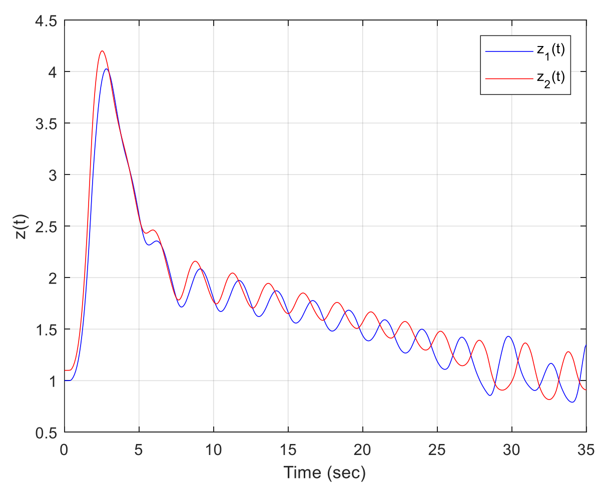

Fig. 7. Time series for $z(t)$ with different initial conditions

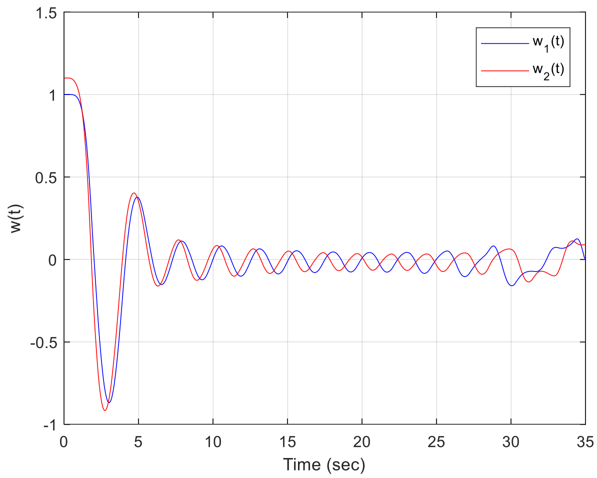

Fig. 8. Time series for $w(t)$ with different initial conditions

$$V(e) = \frac{1}{2}\sum_{i=1}^{4} e_i^2 \tag{10}$$

The first derivative of (10) is given by:

$$\dot{V}(e) = \sum_{i=1}^{4}[e_i \cdot \dot{e}_i] \tag{11}$$

By substituting (9) into (11) and simplifying:

$$\dot{V}(e) = -k_i \sum_{i=1}^{4} e_i^2 \tag{12}$$

If $k_i > 0$ for $i = 1,2,3,4$ then $\dot{V}(e) < 0$, that is, the first derivative of Lyapunov function candidate is negative definite. Consequently, the asymptotic convergence to zero of the synchronization error can be guaranteed from the second theorem of Lyapunov's stability. Now, if $k_i > 1/2$ for $i = 1,2,3,4$ then the following inequality is true:

$$\dot{V}(e) < -V(e) \tag{13}$$

This inequality can be solved as follows:

$$\frac{d}{dt}[V(e)] < -V(e)$$

$$\frac{d[V(e)]}{V(e)} < -dt$$

$$\int_{V(0)}^{V(e)} \frac{d[V(e)]}{V(e)} < -\int_0^t d\tau$$

$$\ln[V(e)]\,|_{V(e(0))}^{V(e(t))} < -\tau|_0^t$$

$$\ln[V(e)] - \ln[V(e(0))] < -t$$

$$\ln\left[\frac{V(e)}{V(e(0))}\right] < -t$$

$$\frac{V(e)}{V(e(0))} < \exp(-t)$$

$$V(e) < V(e(0)) \cdot \exp(-t) \tag{14}$$

From (10), $V(e(0))$ can be expressed as:

$$V(e(0)) = \frac{1}{2}\sum_{i=1}^{4} e_i^2(0) \tag{15}$$

If (10) and (15) are substituted into (14), then:

$$\frac{1}{2}\sum_{i=1}^{4} e_i^2 < \frac{1}{2}\sum_{i=1}^{4} e_i^2(0) \cdot \exp(-t) \tag{16}$$

Since $\|e\| = \sqrt{\sum_{i=1}^{n} e_i^2}$ and $\|e(0)\| = \sqrt{\sum_{i=1}^{n} e_i^2(0)}$, (16) can be expressed as:

$$\frac{1}{2}\|e\|^2 < \frac{1}{2}\|e(0)\|^2 \cdot \exp(-t)$$

$$\|e\|^2 < \|e(0)\|^2 \cdot \exp(-t)$$

By taking square root of both sides of the last inequality,

$$\|e\| < \|e(0)\| \cdot \exp(-0.5 \cdot t) \tag{17}$$

From (17), the exponential convergence to zero of the synchronization error can be guaranteed.

Another way to demonstrate the exponential convergence of the synchronization error is by solving the ordinary first-order differential equation described in (9):

$$\frac{d}{dt}(e_i) = -k_i e_i$$

$$\frac{d(e_i)}{e_i} = -k_i \cdot dt$$

$$\int_{e(0)}^{e(t)} \frac{d(e_i)}{e_i} = -k_i \int_0^t d\tau$$

$$\ln(e_i)\,|_{e(0)}^{e(t)} = -k_i \cdot \tau|_0^t$$

$$\ln[e_i(t)] - \ln[e_i(0)] = -k_i \cdot t$$

$$\ln\left[\frac{e_i(t)}{e_i(0)}\right] = -k_i \cdot t$$

$$\frac{e_i(t)}{e_i(0)} = \exp(-k_i \cdot t)$$

$$e_i(t) = e_i(0) \cdot \exp(-k_i \cdot t) \text{ for } i = 1,2,3,4 \tag{18}$$

From (18), the convergence to zero of the synchronization error can also be verified. However, in this last result, it can be seen how a direct relationship between the value of k_i for $i = 1,2,3,4$ and the speed of the convergence process can be determined.

5. Numerical Simulations

In this section, the performance of the control law (7) is illustrated. For the first test, the control gains values are selected as $k_1 = k_2 = k_3 = k_4 = 1$. Initial condition of master system is (1,1,1,1) whereas the initial condition of slave system is (0.7,0,0,0.7). The synchronization process is shown in figures 9 – 12. As can be appreciated in these figures, the states of the slave system (3) can follow successfully the states of the autonomous master system (2). A better appreciation of the convergence process is presented in figure 13 by means of error evolution $e(t)$. The control inputs are shown in figures 14. It can be observed that these signals could be implemented in a functioning system due to their smoothness and limited range.

As it has been shown in (18), the control gains can modify the speed of the convergence process. This can be illustrated by means of a second simulation test. In this test, the control gains are selected as $k_1 = 250$, $k_2 = 150$, $k_3 = 200$ and $k_4 = 300$.

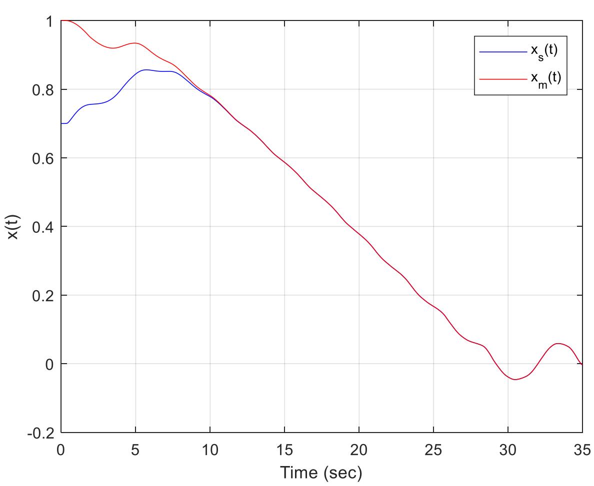

Fig. 9. Convergence process for $x(t)$

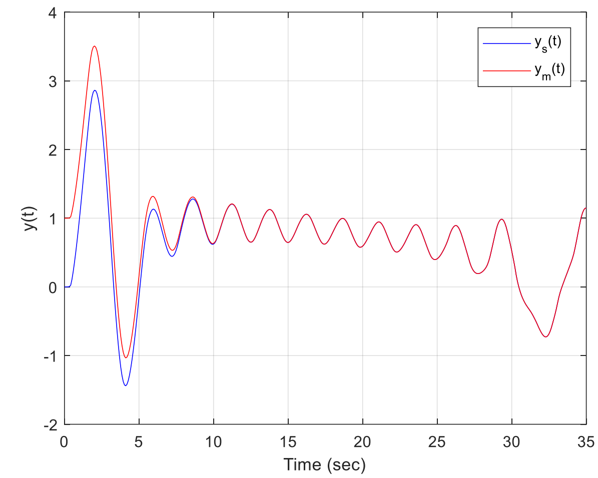

Fig. 10. Convergence process for $y(t)$

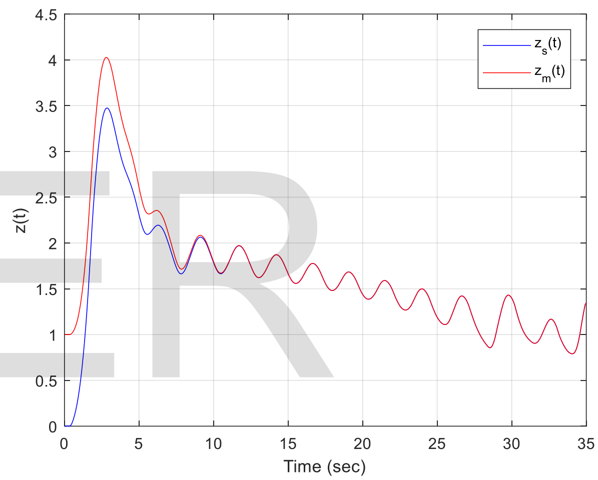

Fig. 11. Convergence process for $z(t)$

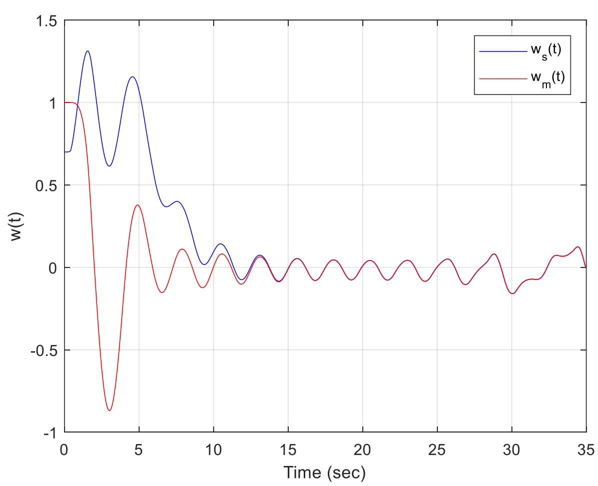

Fig. 12. Convergence process for $w(t)$

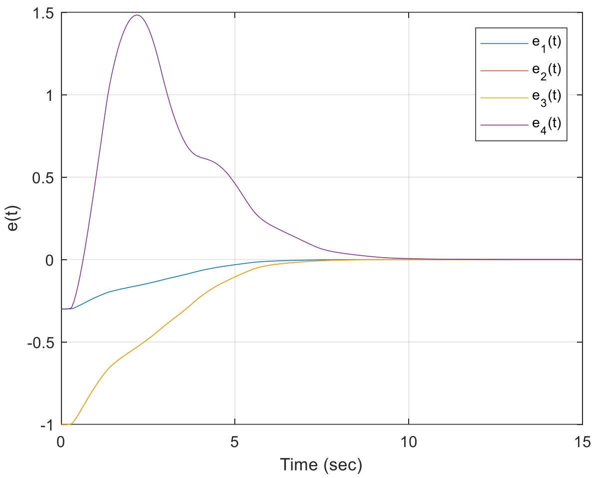

Fig. 13. Error evolution $e(t)$ when $k_1 = k_2 = k_3 = k_4 = 1$.

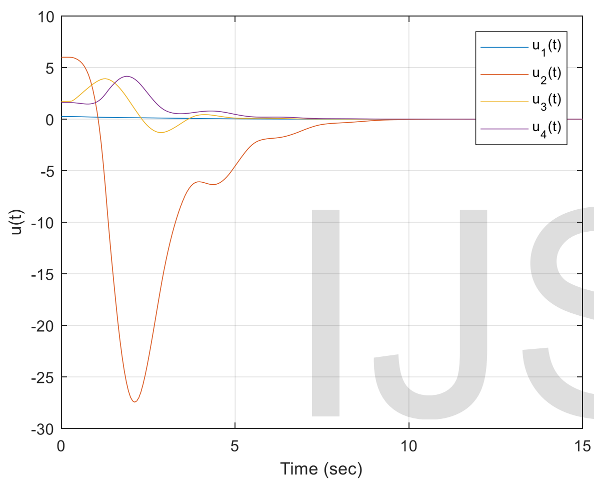

Fig. 14. Control inputs when $k_1 = k_2 = k_3 = k_4 = 1$.

As can be seen from figure 14, the speed of convergence process has considerably been improved. In contrast, from figure 16, it can be appreciated that the magnitude of control signals has also increased. The wide range of values of the control law could cause problems for its implementation.

6. Conclusions

Although the chaotic systems are aperiodic oscillators, they can be synchronized by means of a proper control strategy. In this paper, the use of active control in order to synchronize a slave system with a master system was considered. Both systems are based on Nazarimehr system. The workability of the controller here was verified by means of numeric simulation. If necessary, the values of k_1, k_2, k_3, and k_4 gains can be adjusted to increase the speed of convergence. It should be considered that increasing the values of the control gains increases the magnitude of the control inputs. Thus a compromise between these two opposite characteristics should be established. In order to consider this fact, as a future work, the controller here designed will be optimized.

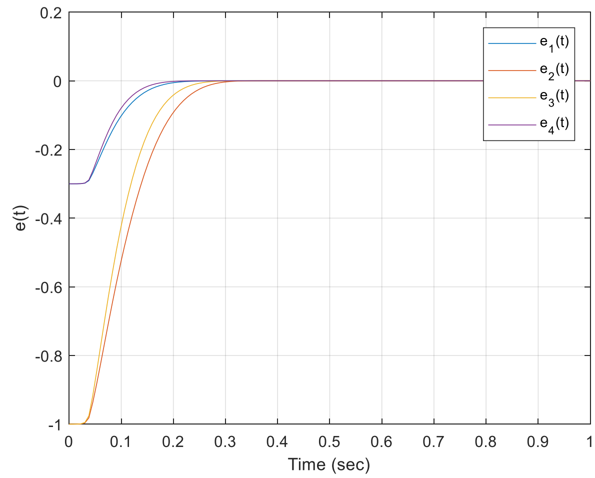

Fig. 15. Error evolution $e(t)$ when $k_1 = 250$, $k_2 = 150$, $k_3 = 200$ and $k_4 = 300$.

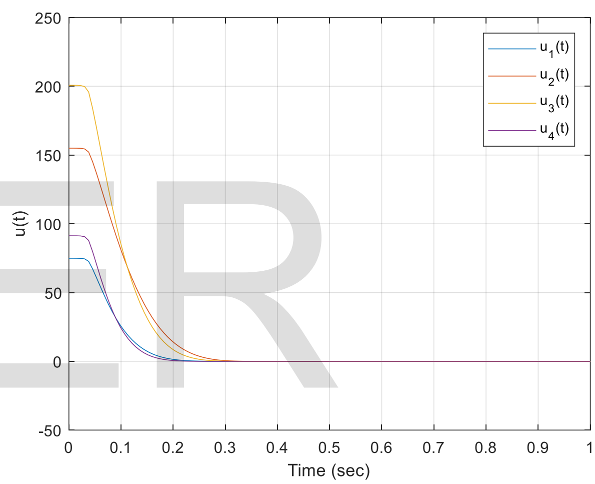

Fig. 16. Control inputs when $k_1 = 250$, $k_2 = 150$, $k_3 = 200$, and $k_4 = 300$.

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